MATH1520AB 2021-22 Quiz 5 (week 9)

Full marks: 10 marks

Time allowed: 15 minutes

1. Find the point on the graph $y = \sqrt{x^2 - 3x + 5}$ that is closest to the point $(1, 0)$.

   Answer.

   A point P on $y = \sqrt{x^2 - 3x + 5}$ can be expressed as $(x, \sqrt{x^2 - 3x + 5})$.

   The square of the distance between P and $(1, 0)$ is $f(x) = (x-1)^2 + (\sqrt{x^2 - 3x + 5} - 0)^2 = 2x^2 - 5x + 6$.

   $f'(x) = 4x - 5 = 0$, $x = \dfrac{5}{4}$. Also, $f''(x) = 4 > 0$.

   Therefore, $(\dfrac{5}{4}, \dfrac{3\sqrt{5}}{4})$ is the point on $y = \sqrt{x^2 - 3x + 5}$ that is closest to the point $(1, 0)$.

2. Consider a cylindrical container of volume $V = 800\ m^3$. Suppose the building cost of the container is directly proportional to its total surface area A (including both the top and bottom). Let r be the base radius and h be the height of the container.

   (a) By using the volume, express the height in terms of the radius.

   (b) Express the total surface area in terms of the radius only.

   (c) Find the base radius that minimizes the building cost in terms of π.

   Answer.

   (a) $V = \pi r^2 h = 800$, $h = \dfrac{800}{\pi r^2}$

   (b) $A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \dfrac{800}{\pi r^2} = 2\pi r^2 + \dfrac{1600}{r}$

   (c) $\dfrac{dA}{dr} = 4\pi r - \dfrac{1600}{r^2} = 0$, $\pi r^3 - 400 = 0$, $r = \sqrt[3]{\dfrac{400}{\pi}}\ m$

   $\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{3200}{r^3}$, $\left.\dfrac{d^2A}{dr^2}\right|_{r=\sqrt[3]{\frac{400}{\pi}}} > 0$

   Therefore, $r = \sqrt[3]{\dfrac{400}{\pi}}\ m$ minimizes the building cost.